R1 Eksamen (LK20), Høst 2022
Løsningsforslag av UDL.no
Del 1

1a) $f(x) = x^4$

$f(x)$ er ikke injektiv (eks. $f(1) = f(-1)$), og har derfor ingen invers.

1b) $g(x) = e^{-(x-2)^2}$, $D_g = [2, \infty)$

$g(x)$ er strengt synkende på $[2, \infty)$, og har derfor en invers

2a) $\lim\limits_{h \to 0} \dfrac{(4+h)^2 - 4^2}{h} = \lim\limits_{h \to 0} \dfrac{8h + h^2}{h} = \lim\limits_{h \to 0} 8 + h = \underline{\underline{8}}$

3) $3\sqrt{11} = \sqrt{99} < \sqrt{100} = 10$ ✓

$10\lg 9 < 10\lg 10 = 10$ ✓

$5\ln 9 = 10\ln 3 > 10\ln e = 10$ ✗

4a) $A(1,1)$, $B(9,7)$, $P(5,9)$

$\vec{AP} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$, $\vec{BP} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$, $\vec{AP} \cdot \vec{BP} = 4(-4) + 8 \cdot 2 = -16 + 16 = 0$

$\Rightarrow \vec{AP} \perp \vec{BP}$

$\Rightarrow \angle APB = 90°$ ■

$\vec{AB} = [8, 6]$

4b) $l: \begin{cases} x = 5 + 8t \\ y = 9 + 6t \end{cases}$ $Q(5+8t, 9+6t)$, $\vec{AQ} = [4+8t, 8+6t]$

$\vec{BQ} = [-4+8t, 2+6t]$

$\vec{AQ} \cdot \vec{BQ} = (4+8t)(-4+8t) + (8+6t)(2+6t)$ $6 \cdot \frac{3}{5} = \frac{18}{5}$

$= \cancel{-16} \cancel{+32t -32t} + 64t^2 + \cancel{16} + 48t + 12t + 36t^2$

$= 100t^2 + 60t = 0$ $\rightarrow Q\left(5 - \frac{24}{5}, 9 - \frac{18}{5}\right)$

$t(100t + 60) = 0 \Rightarrow t = 0 \vee t = -3/5$ $= \underline{\underline{Q(0{,}2, 5{,}4)}}$

$\hookrightarrow$ svarer til P

5) Programmet prøver å finne en tilnærmingsverdi for x slik at $f'(x) = -3$ ($\to x > 1{,}5$)
og deretter uttrykke tangenten i dette punktet.

Ved regning: $f(x) = \frac{6x-3}{x-1}$

$$f'(x) = \frac{(6x-3)'(x-1) - (6x-3)(x-1)'}{(x-1)^2} = \frac{6x-6-6x+3}{(x-1)^2} = -\frac{3}{(x-1)^2}$$

$$-\frac{3}{(x-1)^2} = -3 \Rightarrow (x-1)^2 = 1 \Rightarrow x-1 = 1 \vee x-1 = -1$$
$$x = 2 \vee x = 0$$

$$b = f(2) - f'(2) \cdot 2 = 9 - (-3) \cdot 2 = 9+6 = 15$$

b vil være <u>tilnærmet lik 15</u> mot slutten av programmet.

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Del 2

1a)

$D = (A4, B4)$
$\to (31,\ 93397)$

$E = (A5, B5)$
$\to (40,\ 121848)$

$F = (A6, B6)$
$\to (50,\ 142816)$

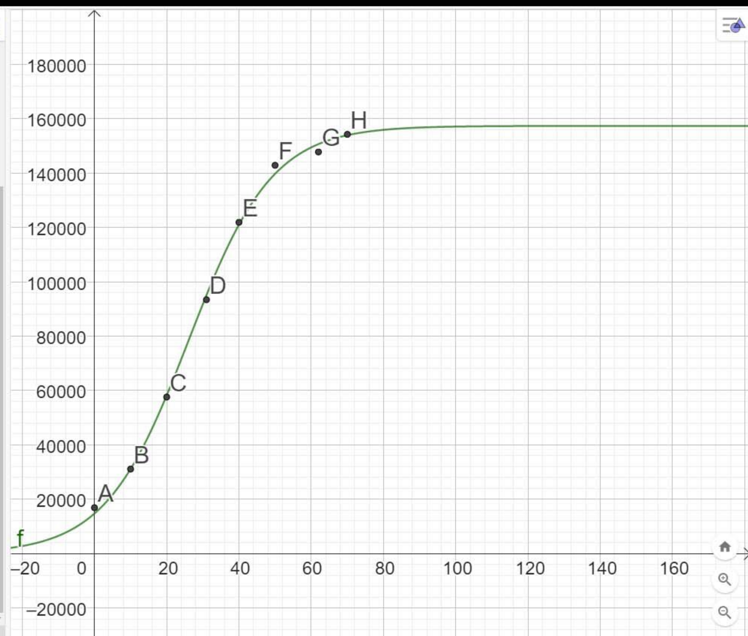
$G = (A7, B7)$
$\to (62,\ 147716)$

$H = (A8, B8)$
$\to (70,\ 154197)$

$lop = \{A, B, C, D, E, F, G, H\}$
$\to \{(0,\ 16924),\ (10,\ 31121),\ (20,$

$f(x) = \text{RegLogist}(lop)$
$$\to \frac{157303.07}{1 + 9.64\ e^{-0.09x}}$$

\+ Skriv inn...



	A	B
1	0	16924
2	10	31121
3	20	57606
4	31	93397
5	40	121848
6	50	142816
7	62	147716
8	70	154197

b)

$f(x) = \text{RegLogist}(lop)$
$$\to \frac{157303.07}{1 + 9.64\ e^{-0.09x}}$$

$l1 = \text{Løs}(f''(x) = 0)$
$$\to \left\{x = \frac{-14788505}{1282467} \ln\left(\frac{175892942422787}{1695531302400860}\right)\right\}$$

$l2 = l1$
$\to \{x = 26.13\}$

$svarB = 1950 + \text{Løsninger}(l2)$
$\to (1976.13)$

Størst vekst i første kvartal av år 1976.

c)

$N = (A6, C6)$
$\to (50,\ 123761)$

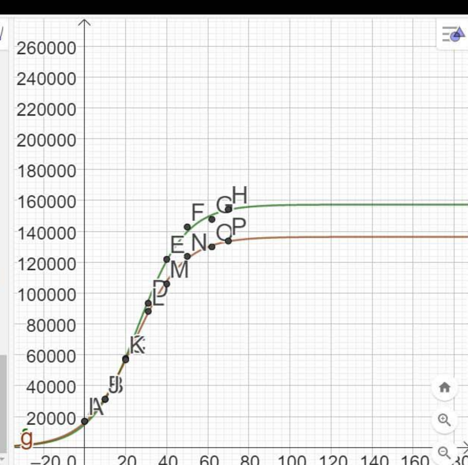
$O = (A7, C7)$
$\to (62,\ 129900)$

$P = (A8, C8)$
$\to (70,\ 133725)$

$l3 = \{I, J, K, L, M, N, O, P\}$
$\to \{(0,\ 16924),\ (10,\ 31253),\ (20,\ 56$

$g(x) = \text{RegLogist}(l3)$
$$\to \frac{136386.59}{1 + 7.44\ e^{-0.08x}}$$

\+ Skriv inn...



	A	B	C
1	0	16924	16924
2	10	31121	31253
3	20	57606	56770
4	31	93397	88168
5	40	121848	105941
6	50	142816	123761
7	62	147716	129900
8	70	154197	133725

Ut fra denne modellen ser det ut som at Norge på sikt vil kunne være selvforsynt med strøm, med en buffer til lagring og/eller eksport.

↳ på ca. 20TWh

2a)

$Q0 = 3.5$

0 50

$Q(t) = Q_0 \left(1 - e^{-2.3t}\right)$

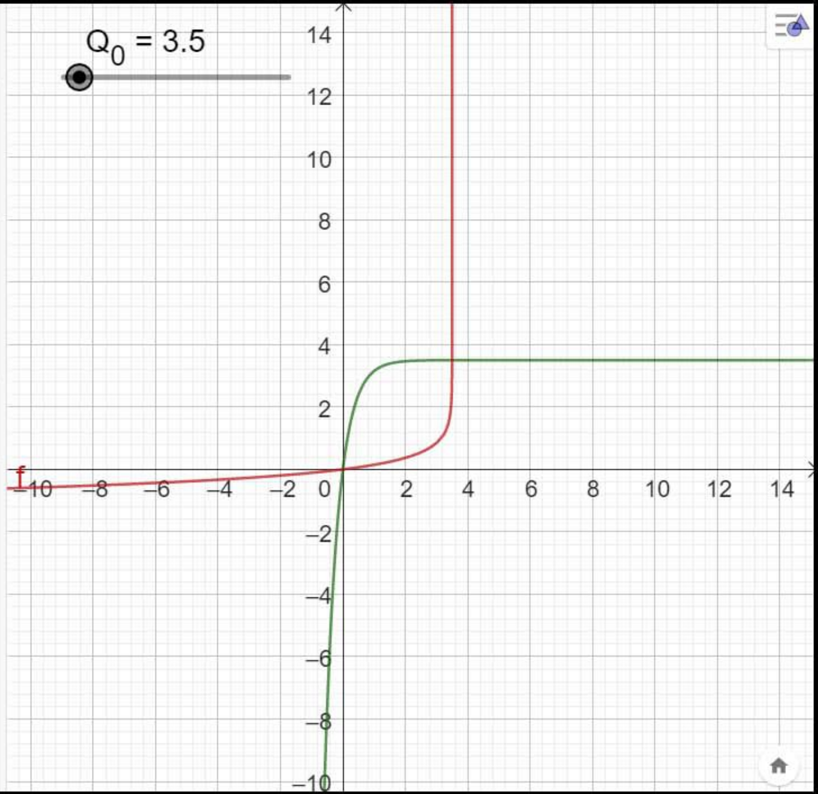
$\rightarrow 3.5 \left(1 - e^{-2.3t}\right)$

$f(t) = \text{Invers}(Q)$

$\rightarrow \dfrac{\log_e\left(1 - \frac{t}{3.5}\right)}{-2.3}$

+ Skriv inn...

$Q^{-1}(t) = \dfrac{\ln\left(1 - t/3{,}5\right)}{-2{,}3}$



b)

$\text{Løs}(Q(t) = 0.9 \cdot Q_0)$

1 $\rightarrow \left\{t = \dfrac{10}{23} \ln(10)\right\}$

2 $1

$\approx \{t = 1\}$

90% ladning på ca. 1s.

3a)

1 $A := (0,0)$
$\rightarrow$ $\mathbf{A := (0,0)}$

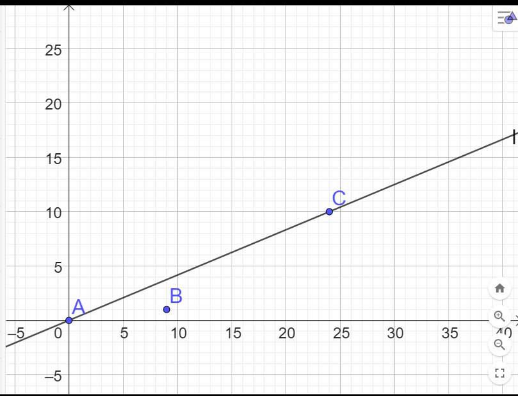
2 $B := (9,1)$
$\rightarrow$ $\mathbf{B := (9,1)}$

3 $C := (24,10)$
$\rightarrow$ $\mathbf{C := (24,10)}$

4 $l := \text{Kurve}(12t, 5t, t, 0, 100)$
$\rightarrow$ $\mathbf{l : 5x - 12y = 0}$

5 $\text{Avstand}(C, l)$
$\rightarrow$ $\mathbf{0}$

6



Avstanden mellom C og l er 0, så C ligger på l. ■

b)

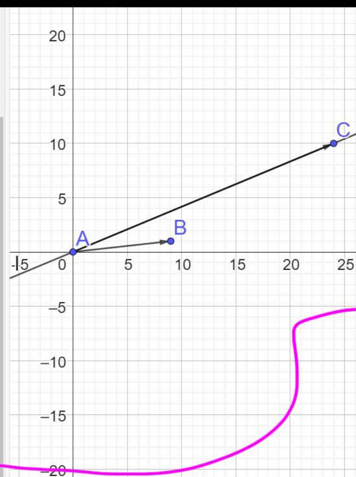
6 $AB := \text{Vektor}(A, B)$
$\rightarrow$ $\mathbf{AB := \begin{pmatrix} 9 \\ 1 \end{pmatrix}}$

7 $AC := \text{Vektor}(A, C)$
$\rightarrow$ $\mathbf{AC := \begin{pmatrix} 24 \\ 10 \end{pmatrix}}$

8 $sp := \text{Skalarprodukt}(AB, AC)$
$\rightarrow$ $\mathbf{sp := 226}$

9 $\text{Løs}(sp = \text{Lengde}(AB)\ \text{Lengde}(AC)\ \cos(x°))$
$\rightarrow$
$$\left\{ x = \frac{360\,k_1\,\pi - 180\,\cos^{-1}\left(113 \cdot \frac{\sqrt{82}}{1066}\right)}{\pi}, x = \ldots \right.$$

10 $\$9$
$\approx$ $\mathbf{\{x = 360\,k_1 - 16.28, x = 360\,k_1 + 16.28\}}$



La $x° = \angle BAC$.
Bruker
$\vec{AB} \cdot \vec{AC} = |\vec{AB}||\vec{AC}| \cos x$

$\rightarrow x \approx \underline{\underline{16{,}28°}}$

c)

8 $D1 := (12\,t, 5\,t)$ D₁ : Arbitrært pkt. på l.
$\rightarrow$ $\mathbf{D1 := (12\,t, 5\,t)}$

9 $AD := \text{Vektor}(A, D1)$
$\rightarrow$ $\mathbf{AD := \begin{pmatrix} 12\,t \\ 5\,t \end{pmatrix}}$

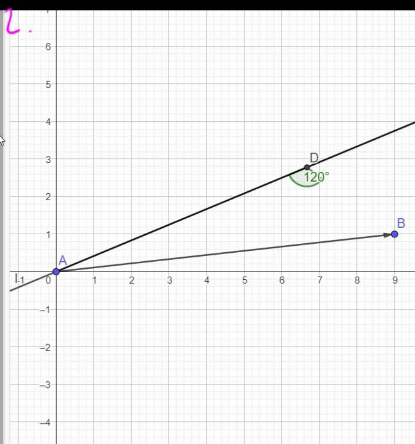
10 $BD := \text{Vektor}(B, D1)$
$\rightarrow$ $\mathbf{BD := \begin{pmatrix} 12\,t - 9 \\ 5\,t - 1 \end{pmatrix}}$

11 $\text{Løs}(\text{Skalarprodukt}(AD, BD) = \text{Lengde}(AD)\ \text{Lengde}(BD)\ \cos(120°))$
$\rightarrow$ $\mathbf{\left\{ t = 0, t = \boxed{\frac{-11\sqrt{3} + 113}{169}} \right\}}$

12 $t1 := \text{HøyreSide}(\text{Element}(\$11, 2))$
$\rightarrow$ $\mathbf{t1 := \frac{-11\sqrt{3} + 113}{169}}$

13 $D := (12\,t1, 5\,t1)$
$\rightarrow$ $\mathbf{D := \left(\frac{-132\sqrt{3} + 1356}{169}, \frac{-55\sqrt{3} + 565}{169} \right)}$ SVAR

14 D
$\approx$ $\mathbf{(6.67, 2.78)}$ Avrunding



d)

15 $E1 := (12\,t, 5\,t)$ → Hjelpevariabel
$\rightarrow$ $\mathbf{E1 := (12\,t, 5\,t)}$

16 $a1 := \text{Avstand}(B, l)$ → Høyda i trekanten
$\rightarrow$ $\mathbf{a1 := \frac{33}{13}}$

17 $AE1 := \text{Vektor}(A, E1)$
$\rightarrow$ $\mathbf{AE1 := \begin{pmatrix} 12\,t \\ 5\,t \end{pmatrix}}$

18 $areal1 := 0.5\ \text{Lengde}(AE1)\ a1$
$\rightarrow$ $\mathbf{areal1 := \frac{33}{2}\,|t|}$

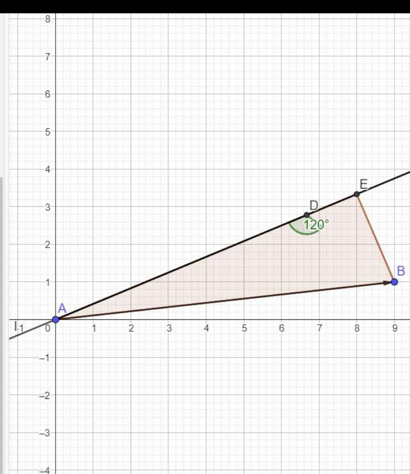
19 $\text{Løs}(areal1 = 11, t)$
$\rightarrow$ $\mathbf{\left\{ t = \frac{-2}{3}, t = \frac{2}{3} \right\}}$

Finner t slik at arealet = 11

20 $t2 := \text{HøyreSide}(\text{Element}(\$19, 2))$ Henter verdien
$\rightarrow$ $\mathbf{t2 := \frac{2}{3}}$

21 $E := (12 \cdot t2, 5 \cdot t2)$
$\rightarrow$ $\mathbf{\underline{E := \left(8, \frac{10}{3}\right)}}$ Endelig pkt. E

22 $areal := \text{Mangekant}(A, B, E)$ Verifikasjon
$\rightarrow$ $\mathbf{areal := 11}$



4a) Moteksempel: La $f(x) = x^2$. $f(2) = f(-2)$ men $2 \neq -2$.

Påstanden holder kun for injektive funksjoner.

b) Sann. $\ln(x)$ er strengt voksende på $(0, \infty)$, så $a > b \Rightarrow \ln a > \ln b$.

c) Sann. $\ln(ax) = \ln a + \ln x = \ln x + \text{konstant}$

så $\frac{d}{dx}\ln(ax) = \frac{d}{dx}(\ln x + \cancel{\text{konstant}}) = \frac{d}{dx}\ln x$

5a)

1 $f(x) := \text{Funksjon}(1 - x^2, 0, 1)$

$\rightarrow$ $f(x) := \text{Dersom}(0 \le x \le 1, 1 - x^2)$

2 $a := 0.5$

$\rightarrow$ $a := \frac{1}{2}$

3 $P := (a, f(a))$

$\rightarrow$ $P := \left(\frac{1}{2}, \frac{3}{4}\right)$

4 $T := \text{Tangent}(a, f)$

$\rightarrow$ $T : y = -x + \frac{5}{4}$

5 $A1 := \text{Skjæring}(T, y = 0)$

$\rightarrow$ $A1 := \left\{\left(\frac{5}{4}, 0\right)\right\}$

6 $B1 := \text{Skjæring}(T, x = 0)$

$\rightarrow$ $B1 := \left\{\left(0, \frac{5}{4}\right)\right\}$

Gir A og B som lister

7 $O := (0, 0)$

$\rightarrow$ $O := (0, 0)$

8 $A := \text{Element}(\$5, 1)$

$\rightarrow$ $A := \left(\frac{5}{4}, 0\right)$

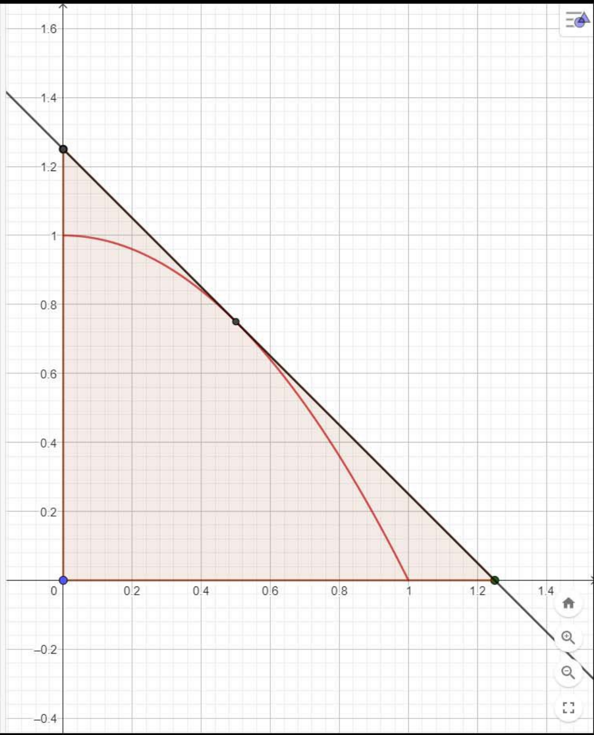
9 $B := \text{Element}(\$6, 1)$

$\rightarrow$ $B := \left(0, \frac{5}{4}\right)$

Henter ut de faktiske punktene

10 $t1 := \text{Mangekant}(O, A, B)$

$\rightarrow$ $t1 := 0.78$



↳ ΔOAB har areal $\underline{\underline{0{,}78}}$

b)

1 $f(x) := 1 - x^2$

$\rightarrow$ $f(x) := -x^2 + 1$

2 $P := (a, f(a))$

$\approx$ $P := (a, -a^2 + 1)$

3 $\text{Tangent}(P, f)$

$\rightarrow$ $y = a^2 - 2\,a\,x + 1$

4 $A(a) := 0.5 \underbrace{\left(a - \frac{f(a)}{f'(a)}\right)}_{\text{bredde}} \overbrace{(1 + a^2)}^{\text{høyde}}$

$\approx$ $A(a) := \frac{0.25\,a^4 + 0.5\,a^2 + 0.25}{a}$

5 $\text{Løs}(A'(a) = 0)$

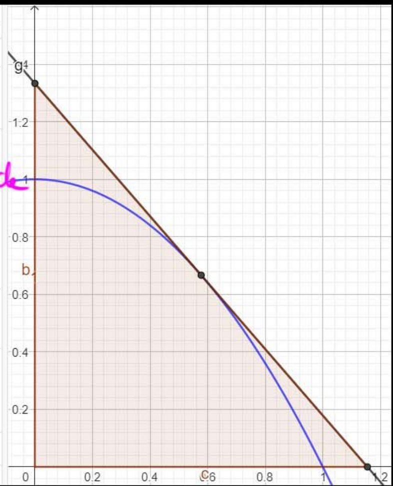
$\approx$ $\{a = -0.58, a = 0.58\}$

6 $A\left(\frac{\sqrt{3}}{3}\right)$

$\approx$ 0.77

minste areal

7



Tangent: $y - y_0 = m(x - x_0)$

$y - f(a) = f'(a)(x - a)$

Bredde: x-verdi når $y = 0$

Løs $0 - f(a) = f'(a)(x - a)$

$x = a - \frac{f(a)}{f'(a)}$

Høyde: y-verdi når $x = 0$

Løs $y - f(a) = f'(a)(0 - a)$

$y = 1 + a^2$

6)

```python
# Henter punktene fra bruker. Fjerner eventuelle mellomrom.
pointAin = input("Punkt 1: ").replace(" ", "").split(",")
pointBin = input("Punkt 2: ").replace(" ", "").split(",")
pointCin = input("Punkt 3: ").replace(" ", "").split(",")

# Gjør om punktene til vektorer som starter i origo
OA = [int(x) for x in pointAin]
OB = [int(x) for x in pointBin]
OC = [int(x) for x in pointCin]

# OA + OB + OC
vectorSum = [
    OA[0] + OB[0] + OC[0],
    OA[1] + OB[1] + OC[1]
]

# Tyndepunktet er en tredel av vektorsummen
OT = [round((1/3)*x, 2) for x in vectorSum]

print(f'Tyndepunkt: ({OT[0]}, {OT[1]})')
```

7a)

$f(x) = 2x + 5 + \frac{1}{x-1}$

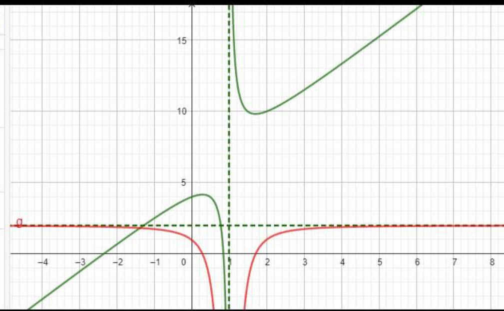
$g(x) = f'(x)$

$\rightarrow \frac{2x^2 - 4x + 1}{x^2 - 2x + 1}$

$l1 = \text{Asymptote}(f'(x))$

$\rightarrow \{y = 2, x = 1\}$

\+ Skriv inn...



$f'(x)$ har verdimengden $V_{f'} = (-\infty, 2)$ så $f'(x) = k$ har løsninger når $k \in (-\infty, 2)$.

b) $f'(x)$ er symmetrisk om vertikalen $x = 1$ så $f'(1-r) = f'(1+r)$ for $r \in \mathbb{R} \setminus \{0\}$. Eks. $f(-2) = f(4) \approx 1,89$.

$a = f'(-2)$

$\rightarrow 1.89$

$b = f'(4)$

$\rightarrow 1.89$

c)

1 $g(x) := a \cdot x + b + \frac{1}{x+d}$

$\rightarrow$ $\mathbf{g(x) := a\,x + b + \frac{1}{d+x}}$

2 $\text{Løs}(g'(x) = 4)$

$\rightarrow \left\{ x = \frac{-(\sqrt{a-4}\, d + 1)}{\sqrt{a-4}}, x = \frac{-(a-4)\, d + \sqrt{a-4}}{a-4} \right\}$

$g'(x) = 4$ er løselig når $a - 4 > 0$

$\underline{\underline{a > 4}}$

d)

3 $g2(x) := \text{ByttUt}(\$1, a = 3)$

$\rightarrow$ $\mathbf{g2(x) := b + 3x + \frac{1}{d+x}}$

4 $g2'(x)$

$\rightarrow \frac{3d^2 + 3x^2 + 6dx - 1}{d^2 + x^2 + 2dx}$

5 $\text{Løs}(g2'(x) = k)$

$\rightarrow \left\{ x = \frac{-(\sqrt{-k+3}\, d + 1)}{\sqrt{-k+3}}, x = \frac{-(k-3)\, d - \sqrt{-k+3}}{k-3} \right\}$

$$x_1 = \frac{-(\sqrt{3-k}\, d + 1)}{\sqrt{3-k}} = -d - \frac{1}{\sqrt{3-k}}$$

$$x_2 = \frac{-(k-3)d - \sqrt{3-k}}{k-3} = -d + \frac{1}{\sqrt{3-k}}$$

$$\Rightarrow x = -d \pm \frac{1}{\sqrt{3-k}}$$

Ser at $g'(x) = k$ nå har løsninger som er symmetrisk om $x = -d$.

Pålagt at $3 - k \geq 0 \Rightarrow k \leq 3$

e)

6 $g2'(-1) = g2'(5)$

$\rightarrow$ $\mathbf{\frac{3d^2 - 6d + 2}{d^2 - 2d + 1} = \frac{3d^2 + 30d + 74}{d^2 + 10d + 25}}$

7 $g2(1) = 7$

$\rightarrow$ $\mathbf{b + 3 + \frac{1}{d+1} = 7}$

8 $\text{Løs}(\{\$6, \$7\}, \{b, d\})$

$\rightarrow$ $\mathbf{\{\{b = 5, d = -2\}\}}$

$\underline{\underline{b = 5,\ d = -2}}$ løser likningssettet.