Fasit til øvingsoppgaver til eksamen

Oppgave 1

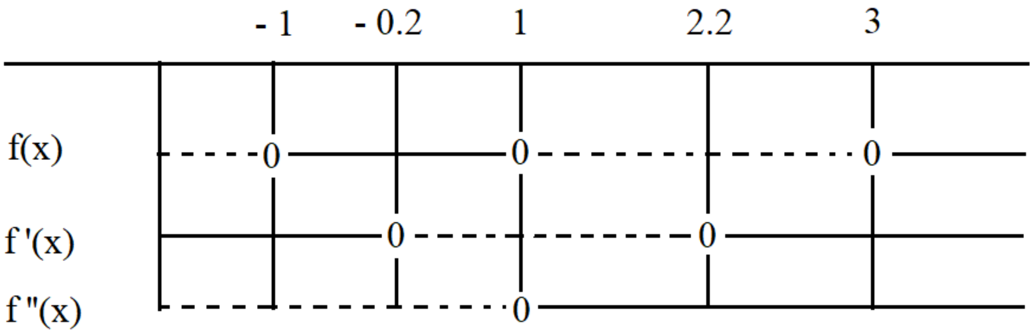
a)

$g(x) = 0 \; for \; x \approx -0.2 \; og \; x \approx 2.2. \; f(x) \; har \; toppunkt \; og \; bunnpunkt \; for \; disse \; verdiene. \; g(x) \; er \; derfor \; den \; deriverte \; av \; f(x).$

$h(x) = 0 \; for \; x \approx 1 \; og \; g(x) \; har \; bunnpunkt \; for \; samme \; x - verdi. \; h(x) \; er \; derfoe \; den \; deriverte \; av \; g(x)$

$f(x) \; er \; derfor \; den \; oprinnelige \; funksjonen, g(x) den \; deriverte \; og \; h(x) \; den \; dobbeltderiverte.$

b)



Oppgave 2

a)

Stigningstall $a = -1$ og skjæringspunkt med y – aksen y = 1

$g(x) = -x + 1$

b)

$$A = \int_0^1 \big(f(x) - g(x)\big)dx = \int_0^1 \big(e^x - (-x+1)\big)dx = \int_0^1 (e^x + x - 1)dx =$$

$$\left[e^x + \frac{1}{2}x^2 - x\right]_0^1 = e^1 + \frac{1}{2}\cdot 1^2 - 1 - \left(e^0 + \frac{1}{2}\cdot 0^2 - 0\right) = e + \frac{1}{2} - 1 - 1 = \underline{\underline{e - \frac{3}{2}}}$$

c)

$$\pi \int_0^1 \left(f(x)^2 - g(x)^2\right) dx = \pi \int_0^1 \left((e^x)^2 - (-x+1)^2\right) dx =$$

$$\pi \int_0^1 (e^{2x} - (x^2 - 2x + 1)\,dx = \pi \left[\frac{1}{2}e^{2x} - \frac{1}{3}x^3 + x^2 - x\right]_0^1 =$$

$$\pi \left(\frac{1}{2}e^{2\cdot 1} - \frac{1}{3}\cdot 1^3 + 1^2 - 1 - \left(\frac{1}{2}e^{2\cdot 0} - \frac{1}{3}\cdot 0^3 + 0^2 - 0\right)\right) =$$

$$\pi \left(\frac{1}{2}e^2 - \frac{1}{3} + 1 - 1 - \frac{1}{2}\right) = \underline{\underline{\pi \left(\frac{1}{2}e^2 - \frac{5}{6}\right)}}$$

Oppgave 3

a)

$$k = \frac{a_2}{a_1} = \underline{\underline{\frac{1-2x}{2x}}}$$

b)

$$-1 < \frac{1-2\text{x}}{2\text{x}} < 1$$

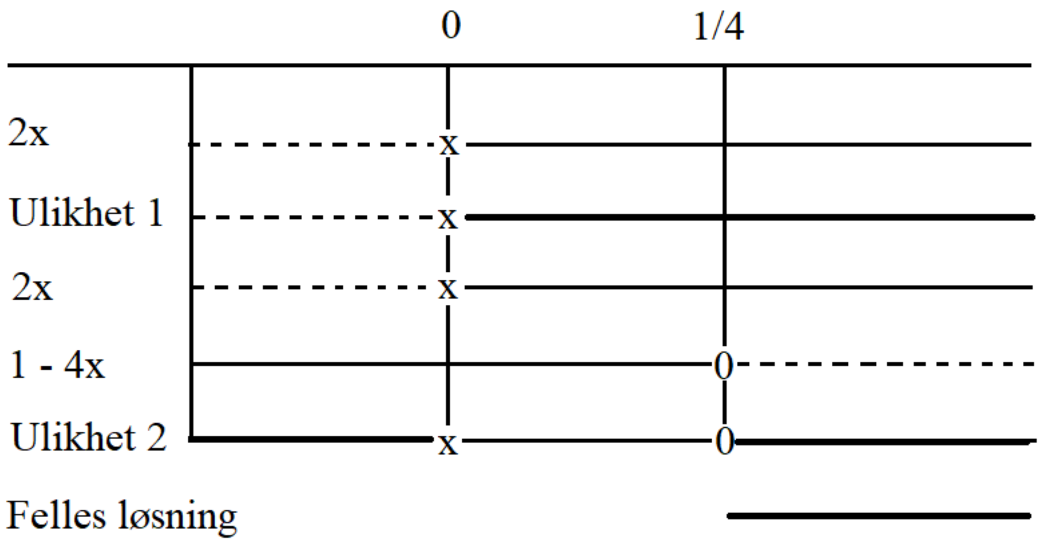
$$-1 < \frac{1-2\text{x}}{2\text{x}} \qquad\qquad \frac{1-2\text{x}}{2\text{x}} < 1$$

$$0 < \frac{1-2\text{x}}{2\text{x}} + 1 \qquad\qquad \frac{1-2\text{x}}{2\text{x}} - 1 < 0$$

$$0 < \frac{1-2\text{x}}{2\text{x}} + \frac{2x}{2x} \qquad\qquad \frac{1-2\text{x}}{2\text{x}} - \frac{2x}{2x} < 0$$

$$0 < \frac{1-2\text{x}+2\text{x}}{2\text{x}} \qquad\qquad \frac{1-2\text{x}-2\text{x}}{2\text{x}} < 0$$

$$0 < \frac{1}{2\text{x}} \qquad\qquad \frac{1-4\text{x}}{2\text{x}} < 0$$



$$\underline{\underline{Konvergensintervallet:\ \ x \epsilon \left\langle \frac{1}{4}, \rightarrow \right\rangle}}$$

c)

$$s = \frac{a_1}{1-k} = \frac{2x}{1 - \frac{1-2x}{2x}} = \frac{2x \cdot 2x}{\left(1 - \frac{1-2x}{2x}\right) \cdot 2x} = \frac{4x^2}{2x - (1-2x)} = \underline{\underline{\frac{4x^2}{4x-1}}}$$

d)

$$\frac{4x^2}{4x-1} = 1$$

$$4x^2 = 4x - 1$$

$$4x^2 - 4x + 1 = 0$$

$$\underline{\underline{x = \frac{1}{2}}}\ \ (Denne\ verdien\ er\ med\ i\ konvergensintervallet\ og\ er\ derfor\ en\ riktig\ løsning)$$

Oppgave 4.

a)

$$\overrightarrow{AB} = \underline{\underline{[-1, 2, 1]}} \quad \overrightarrow{BC} = [0, -3, 1] \quad \overrightarrow{AC} = [-1, -1, 2]$$

$$3\overrightarrow{AB} - \frac{1}{2}\overrightarrow{BC} = 3 \cdot [-1, 2, 1] - \frac{1}{2} \cdot [0, -3, 1] = \underline{\underline{\left[-3, \frac{15}{2}, \frac{5}{2}\right]}}$$

b)

$$\left|\overrightarrow{AB}\right| = \sqrt{(-1)^2 + 2^2 + 1^2} = \underline{\underline{\sqrt{6}}}$$

$$\left|\overrightarrow{AC}\right| = \sqrt{(-1)^2 + (-1)^2 + 2^2} = \underline{\underline{\sqrt{6}}}$$

$$\left|\overrightarrow{BC}\right| = \sqrt{0^2 + (-3)^2 + 1^2} = \underline{\underline{\sqrt{10}}}$$

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = (-1) \cdot (-1) + 2 \cdot (-1) + 1 \cdot 2 = 1$$

$$\overrightarrow{BC} \cdot \overrightarrow{BA} = 0 \cdot 1 + (-3) \cdot (-2) + 1 \cdot (-1) = 5$$

$$\cos \angle A = \frac{1}{\sqrt{6} \cdot \sqrt{6}} = \frac{1}{6} \rightarrow \underline{\underline{\angle A = 80.4^{o}}}$$

Siden det er likebeint trekant er de 2 andre vinklene

$$\angle B = \frac{1}{2}(180^{o} - 80.4^{o}) = \underline{\underline{49.8^{o}}}$$

$$\angle C = \underline{\underline{49.8^{o}}}$$

c)

$$A = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}|[-1,2,1] \times [-1,-1,2]| = \frac{1}{2}\begin{vmatrix} \overrightarrow{e_x} & \overrightarrow{e_y} & \overrightarrow{e_z} \\ -1 & 2 & 1 \\ -1 & -1 & 2 \end{vmatrix}$$

$$\frac{1}{2}\left|\left[2 \cdot 2 - 1(-1), -\big(-1 \cdot 2 - 1(-1)\big), (-1)(-1) - 2(-1)\right]\right| = \frac{1}{2}|[5,1,3]|$$

$$\frac{1}{2}\sqrt{5^2 + 1^2 + 3^2} = \underline{\underline{\frac{\sqrt{35}}{2}}}$$

d)

Bruker punktet $A(1,1,0)$ og vektorene $\overrightarrow{AB}$ og $\overrightarrow{AC}$

$$\beta = \begin{cases} x = 1 + (-1)t + (-1)s \\ y = 1 + 2t + (-1)s \\ z = 0 + 1t + 2s \end{cases} = \begin{cases} x = 1 - t - s \\ y = 1 + 2t - s \\ z = t + 2s \end{cases}$$

e)

$$\overrightarrow{BD} = [0 - 0, 0 - 3, z - 1] = [0, -3, z - 1]$$

$$\overrightarrow{AB} \cdot \overrightarrow{BD} = 0 \cdot 1 + (-3) \cdot 2 + (z - 1) \cdot 1 = z - 7 \rightarrow z - 7 = 0 \rightarrow z = 7$$

$$D : \underline{\underline{(0,0,7)}}$$

f)

$$V = \frac{1}{6}\begin{vmatrix} \overrightarrow{OA} \\ \overrightarrow{OB} \\ \overrightarrow{OC} \end{vmatrix} = \frac{1}{6}\begin{vmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{vmatrix} = \frac{1}{6}(1(6-0)-1(0-0)+0(0-0)) \underline{\underline{=1}}$$

g)

$$\vec{n} = \overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \overrightarrow{e_x} & \overrightarrow{e_y} & \overrightarrow{e_z} \\ 1 & 1 & 0 \\ 0 & 3 & 1 \end{vmatrix} = [1 \cdot 1 - 0 \cdot 3, -(1 \cdot 1 - 0 \cdot 0), 1 \cdot 3 - 1 \cdot 0] = [1, -1, 3]$$

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

$$1(x - 0) - 1(y - 0) + 3(z - 0) = 0$$

$$\underline{\underline{Likning\ for\ planet\ \alpha: \quad x - y + 3z = 0}}$$

Oppgave 5

a)

$$V = grunnflate \cdot høyde$$

$$6.0 = x \cdot 1.5 \cdot h$$

$$h = \frac{6.0}{1.5 \cdot x} = \underline{\underline{\frac{4}{x}}}$$

b)

$$O = F(x) = 2 \cdot x \cdot 1.5 + 2 \cdot x \cdot \frac{4}{x} + 2 \cdot 1.5 \cdot \frac{4}{x} = \underline{\underline{3x + 8 + \frac{12}{x}}}$$

c)

$$F(x) = 3x + 8 + \frac{12}{x} = 3x + 8 + 12 \cdot x^{-1}$$

$$F'(x) = 3 + 0 + (-1)x^{-1-1} = 3 - x^{-2} = 3 - \frac{12}{x^2}$$

$$F'(x) = 3 - 12 \cdot x^{-2}$$

$$F''(x) = -12 \cdot (-2) \cdot x^{-2-1} = 24 \cdot x^{-3} = \frac{24}{x^3}$$

$$F'(x) = 3 - \frac{12}{x^2} = 0$$

$$3 = \frac{12}{x^2}$$

$$3x^2 = 12$$

$$x^2 = \frac{12}{3} = 4$$

$\mathrm{x} = \pm\sqrt{4} = \pm 2 \qquad -2\ ikke\ løsning\ fordi\ siden\ i\ en\ kasse\ må\ være\ positiv.$

$$F''(2) = \frac{24}{2^3} = 3 > 0\ \ Minpunkt$$

$$\underline{\underline{x = 2\,m\ \ gir\ F(x)\ minst\ mulig}}$$

d)

$$O = F(3) = 3 \cdot 2 + 8 + \frac{12}{2} = 20$$

$$\underline{\underline{Overflaten\ blir\ 20\ m^2}}$$

Oppgave 6

a)

$$\int (3 - x^2)\,dx = \underline{\underline{3x - \frac{1}{3}x^3 + C}}$$

b)

$$\int x \cdot lnx\,dx = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} \cdot dx = \frac{1}{2}x^2 \cdot \ln x - \frac{1}{2}\int x \cdot dx =$$

$$\frac{1}{2}x^2 \cdot \ln x - \frac{1}{2} \cdot \frac{1}{2} \cdot x^2 + C = \underline{\underline{\frac{1}{2}x^2\left(\ln x - \frac{1}{2}\right) + C}}$$

$$v = \ln x \qquad v' = \frac{1}{x}$$

$$u = \frac{1}{2}x^2 \qquad u' = x$$

c)

$$\int (2x+2)\sqrt{x^2+2x}\,dx = \int (2x+2)\sqrt{u}\,\frac{du}{2x+2} = \int u^{\frac{1}{2}}\,du = \frac{1}{\frac{1}{2}+1}u^{\frac{1}{2}+1} + C =$$

$$\frac{1}{\frac{3}{2}}u^{\frac{3}{2}} + C = \frac{2}{3}\left(u^{1+\frac{1}{2}}\right) + C = \frac{2}{3}u\sqrt{u} + C \underline{\underline{= \frac{2}{3}(x^2 + 2x)\sqrt{x^2 + 2x} + C}}$$

$$u = x^2 + 2x$$

$$\frac{du}{dx} = 2x + 2$$

$$dx = \frac{du}{2x + 2}$$

d)

$$\int_{-2}^{-1} \frac{4x - 4}{x^2 - 2x}\, dx = \int_{-2}^{-1} \frac{2(2x - 2)}{x^2 - 2x}\, dx = 2\int_{-2}^{-1} \frac{(2x - 2)}{x^2 - 2x}\, dx = 2[\ln|x^2 - 2x|]\,{}_{-2}^{-1}$$

$$2\ln|(-1)^2 - 2(-1)| - 2\ln|(-2)^2 - 2(-2)| = 2\ln 3 - 2\ln 8 \underline{\underline{= 2\ln 3 - 6\ln 2}}$$

e)

$$\int \frac{x - 2}{x^2 + x - 2}\, dx = \int \left(\frac{A}{x - 1} + \frac{B}{x + 2}\right) dx = \int \left(\frac{-\frac{1}{3}}{x - 1} + \frac{\frac{4}{3}}{x + 2}\right) dx$$

$$\frac{x - 2}{(x - 1)(x + 2)} = \frac{A}{x - 1} + \frac{B}{x + 2} \qquad \underline{\underline{= -\frac{1}{3}\ln|x - 1| + \frac{4}{3}\ln|x + 2|}}$$

$$A = \frac{1 - 2}{3} = -\frac{1}{3}$$

$$B = \frac{-2 - 2}{-2 - 1} = \frac{4}{3}$$

Oppgave 7

$$V = \pi \int_{-1}^{0} \left(\frac{2}{x - 1}\right)^2 dx = \pi \int_{-1}^{0} \frac{4}{(x - 1)^2}\, dx = \left[-4\pi \frac{1}{x - 1}\right]_{-1}^{0} =$$

$$-4\pi\left(\frac{1}{0 - 1} - \frac{1}{-1 - 1}\right) \underline{\underline{= 2\pi}}$$

Oppgave 8

$y' = 4x \cdot y \qquad\qquad y = 2 \ \ når \ \ x = 0$

$$\frac{dy}{dx} = 4x \cdot y$$

$$\frac{dy}{y} = 4x \cdot dx$$

$$\int \frac{1}{y} dy = \int 4x \cdot dx$$

$$\ln|y| = 4 \cdot \frac{1}{2}x^2 + C_1$$

$$e^{\ln|y|} = e^{2x^2 + C_1}$$

$$|y| = e^{C_1} \cdot e^{2x^2}$$

$$y = \pm e^{C_1} \cdot e^{2x^2}$$
$y = C \cdot e^{2x^2} \qquad\qquad y = 2 \quad når \ \ x = 0$

$$2 = C \cdot e^{2 \cdot 0^2}$$

$$C = 2$$

$$\underline{\underline{y = 2 \cdot e^{2x^2}}}$$

Oppgave 9

a)
$Amplitude = 2$

$Likevektslinje = -1$

$$Periode = \frac{2\pi}{k} = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

b)
$$f(x) = 2\sin\left(\frac{0}{2}\right) - 1 = -1$$

$\underline{\underline{Skjæringspunkt\ med\ y - aksen\text{:}\ \ (0, -1)}}$

$$f(x) = 2\sin\left(\frac{x}{2}\right) - 1 = 0$$

$$\sin\frac{x}{2} = \frac{1}{2}$$

$$\frac{x}{2} = \frac{\pi}{6} + k \cdot 2\pi$$

$$\frac{x}{2} = \pi - \frac{\pi}{6} + k \cdot 2\pi$$

$$\frac{x}{2} = \frac{\pi}{6} + k \cdot 2\pi$$

$$\frac{x}{2} = \frac{5\pi}{6} + k \cdot 2\pi$$

$$x = \frac{\pi}{6} \cdot 2 + k \cdot 2\pi \cdot 2$$

$$x = \frac{5\pi}{6} \cdot 2 + k \cdot 2\pi \cdot 2$$

$$x = \frac{\pi}{3} + k \cdot 4\pi$$

$$x = \frac{5\pi}{3} + k \cdot 4\pi$$

$$x = \frac{\pi}{3} + 0 \cdot 4\pi = \frac{\pi}{3}$$

$$x = \frac{5\pi}{3} + 0 \cdot 4\pi = \frac{5\pi}{3}$$

$$\underline{\underline{Skjæringspunkter\ med\ x - aksen:\ \left(\frac{\pi}{3}, 0\right)\ og\ \left(\frac{5\pi}{3}, 0\right)}}$$

c)

$$f'(x) = \frac{1}{2} \cdot 2\cos\frac{x}{2} = \cos\frac{x}{2}$$

$$f''(x) = -\frac{1}{2}\sin\frac{x}{2}$$

$$f'(x) = \cos\frac{x}{2} = 0$$

$$\frac{x}{2} = \frac{\pi}{2} + k \cdot 2\pi$$

$$\frac{x}{2} = 2\pi - \frac{\pi}{2} + k \cdot 2\pi$$

$$\frac{x}{2} = \frac{\pi}{2} + k \cdot 2\pi$$

$$\frac{x}{2} = \frac{3\pi}{2} + k \cdot 2\pi$$

$$x = \pi + k \cdot 4\pi$$

$$x = 3\pi + k \cdot 4\pi$$

$$x = \pi + 0 \cdot 4\pi = \pi \qquad Eneste\ verdi\ i\ D_f$$

$$f(\pi) = 2\sin\left(\frac{\pi}{2}\right) - 1 = 1$$

$$f''(\pi) = -\frac{1}{2}\sin\frac{\pi}{2} = -\frac{1}{2} < 0\ \ makspunkt$$

$$\underline{\underline{Makspunkt: (\pi, 1)}}$$