Oppgave 1

DEL 1

a) $f'(x) = \underline{\underline{-\pi \sin(\pi x - 2)}}$

b) $g'(x) = u'v + uv' = \underline{\underline{\sin x + x \cdot \cos x}}$

Oppgave 2

a) $\int (4x^2 + 3x)\,dx = \underline{\underline{\frac{4}{3}x^3 + \frac{3}{2}x^2 + C}}$

b) $\int \overset{v'}{4x^2} \cdot \overset{u}{\ln x}\,dx = uv - \int u'v\,dx$

$= \frac{4}{3}x^3 \cdot \ln x - \frac{4}{3}\int x^2\,dx = \underline{\underline{\frac{4}{3}x^3 \cdot \ln x - \frac{4}{9}x^3 + C}}$

c) $\int_0^{\sqrt{12}} \frac{2x}{x^2+4}\,dx = \int_{x=0}^{x=\sqrt{12}} \frac{\cancel{2x}}{u}\frac{du}{\cancel{2x}} = \ln|u|\Big|_{x=0}^{x=\sqrt{12}} = \ln|x^2+4|\Big|_0^{\sqrt{12}} = \ln(16) - \ln(4)$

$= \underline{\underline{\ln(4)}}$

Oppgave 3

$\left.\begin{matrix} a_1 + a_2 + a_3 + a_4 + a_5 \\ a_1 + 4 + a_3 + a_4 + 13 \end{matrix}\right\} \Rightarrow 4 + 3d = 13$

$+d \quad +d \quad +d \quad +d$

$3d = 9$

$\underline{d = 3} \Rightarrow \underline{a_1 = 1}$

$\Rightarrow \underline{a_n} = 1 + (n-1)\cdot 3 = \underline{3n - 2} \Rightarrow S_n = \frac{n(a_1 + a_n)}{2} = \underline{\underline{\frac{n(3n-1)}{2}}}$

Oppgave 4

a) $\dfrac{dy}{dx} = (\sin x) \cdot y^2$

$\int \frac{1}{y^2}\,dy = \int \sin x \,dx$

$-\frac{1}{y} = -\cos x + C_1$

$\underline{\underline{y = \frac{1}{\cos x + C}}}$

b) $y(\pi) = \frac{1}{-1 + C} = 1 \Rightarrow \underline{C = 2}$

$\underline{\underline{y = \frac{1}{\cos x + 2}}}$

Oppgave 5

a) Nullpunktene (og avgrensningen) er $\underline{x=-1}$ og $\underline{x=1}$

$$\underline{A} = \int_{-1}^{1} (1-x^2)\,dx = \left(x - \frac{1}{3}x^3\right)\Big|_{-1}^{1} = 2\cdot\left(1-\frac{1}{3}\right) = \underline{\underline{\frac{4}{3}}}$$

b) $$V = \pi \int_{-1}^{1} (1-x^2)^2\,dx$$

$$= \pi \int_{-1}^{1} (1-2x^2+x^4)\,dx = \pi\left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right)\Big|_{-1}^{1}$$

$$= 2\pi\left(1-\frac{2}{3}+\frac{1}{5}\right)$$

$$= 2\pi \cdot \frac{15-10+3}{15} = \underline{\underline{\frac{16\pi}{15}}}$$

Oppgave 6

a) Topppunkter når

$$\sin\left(\frac{\pi}{2}(x-1)\right) = 1 \Rightarrow \underline{x=2} \lor \underline{x=6}$$

$$\Rightarrow \underline{\underline{(2,2)}} \lor \underline{\underline{(6,2)}}$$

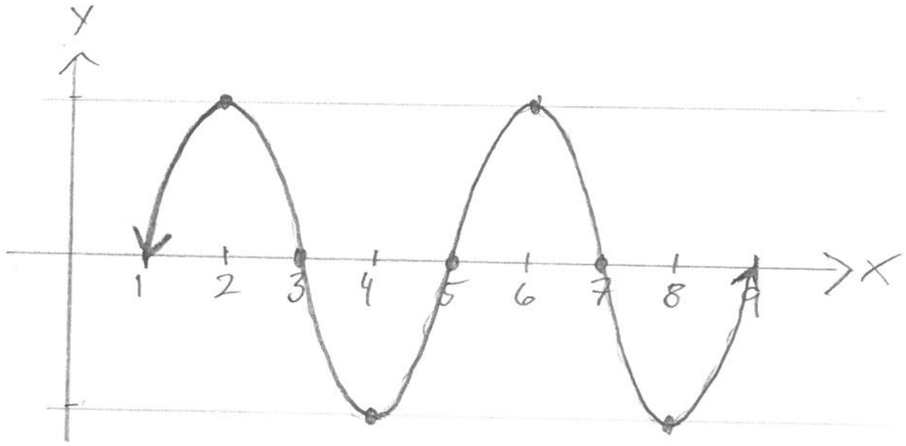
Bunnpunkter når

$$\sin\left(\frac{\pi}{2}(x-1)\right) = -1 \Rightarrow \underline{x=4} \lor \underline{x=8}$$

$$\Rightarrow \underline{\underline{(4,-2)}} \lor \underline{\underline{(8,-2)}}$$

b) Nullpunktene er på likevektslinja, som er midt mellom topp- og bunnpunktene: $\underline{\underline{(3,0)}} \lor \underline{\underline{(5,0)}} \lor \underline{\underline{(7,0)}}$

c)



d) $$\frac{\cancel{2}\sin\left(\frac{\pi}{2}(x-1)\right)}{\cancel{2}} = \frac{\sqrt{3}}{2}$$

$$\frac{\pi}{2}(x-1) = \frac{\pi}{3} \lor \frac{2\pi}{3} \lor \ldots$$

$$x = \underline{\underline{\frac{5}{3}}} \lor \underline{\underline{\frac{7}{3}}} \lor \underline{\underline{\frac{17}{3}}} \lor \underline{\underline{\frac{19}{3}}}$$

(De to siste løsningene får jeg ved å legge til en periode, $p = 4 = \frac{12}{3}$)

Oppgave 7

a) $x^2-6x\underline{+9}+y^2+4y\underline{+4}+z^2-8z\underline{+16}=20\underline{+9}\underline{+4}\underline{+16}$

$$(x-3)^2+(y+2)^2+(z-4)^2=7^2$$

Altså er $\underline{S(3,-2,4)}$ og radien er $\underline{\underline{r=7}}$

b)

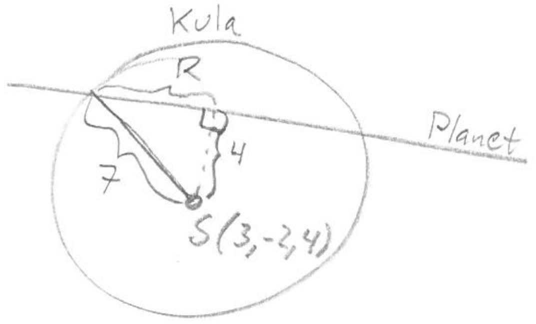
$$\underline{\text{Avstand}}=\frac{|ax+by+cz+d|}{\sqrt{a^2+b^2+c^2}}=\frac{|6\cdot 3-3\cdot(-2)+2\cdot 4-4|}{\sqrt{36+9+4}}$$

$$=\frac{18+6+8-4}{\sqrt{49}}$$

$$=\frac{28}{7}$$

$$=\underline{\underline{4}}$$

c)



Pytagoras gir

$$4^2+R^2=7^2$$

$$R=\sqrt{49-16}=\underline{\sqrt{33}}$$

$$\Rightarrow \text{Arealet}=\pi\cdot R^2=\underline{\underline{33\pi}}$$

Oppgave 8

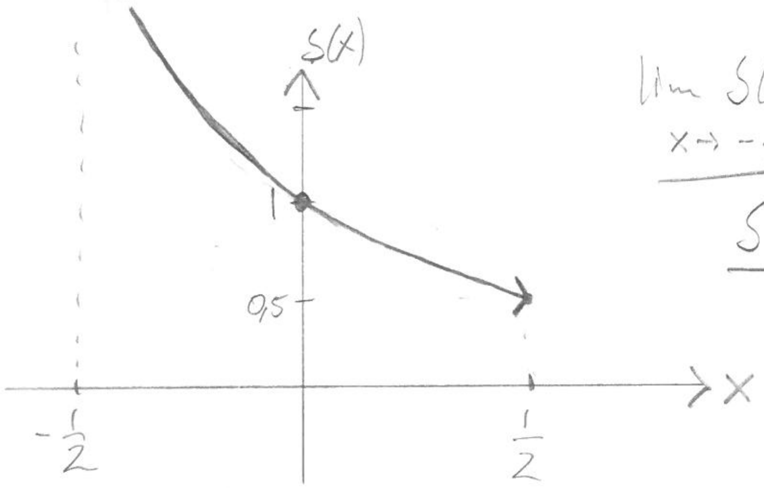
a) Kvotienten er $k = -2x$

$$\Rightarrow |-2x| < 1 \Rightarrow \underline{\underline{-\frac{1}{2} < x < \frac{1}{2}}} \Leftrightarrow \underline{\underline{|x| < \frac{1}{2}}}$$

b) $S(x) = \dfrac{1}{1+2x} \qquad x \in \langle -\frac{1}{2}, \frac{1}{2} \rangle$

(Denne kan løses på flere måter)

Jeg tegner en skisse av $S(x)$:



$$\lim_{x \to -\frac{1}{2}^+} S(x) \to \infty$$

$$\underline{S(0) = 1}$$

$$\underline{\lim_{x \to \frac{1}{2}} S(x) = \frac{1}{2}}$$

Ser at $S(x) = a$ har løsninger for $\underline{\underline{\frac{1}{2} < a < \infty}}$

Oppgave 1

DEL 2

a) Se vedlegg

b) Areal $= \frac{125}{24} \approx 5{,}21$. Se vedlegg. Linje 1-2 i CAS.

c) $T\left(\frac{3}{4}, \frac{25}{8}\right)$. Se vedlegg. Linje 3-5 i CAS.

Oppgave 2

a) Areal $= \frac{13}{2}$. Se vedlegg. Linje 1-5 i CAS.

b) $t = 1{,}8 \lor t = 7{,}85 \lor t = 9{,}29$. Se vedlegg. Linje 6 i CAS.

c) $t = 7$. Se vedlegg.
Jeg lager et uttrykk for volumet (linje 7 i CAS), tegner grafen (se grafikkfelt) og bruker verktøyet Ekstremalpunkt til å finne toppunktet.

Oppgave 3

a) $\underline{\underline{y' = k\underbrace{(12000 - y)}}}$

vekstfarten

Folk som enda ikke er smittet

b) $\frac{dy}{dt} = k(12000 - y)$

$\int \frac{1}{12000 - y} \, dy = \int dt$

$-\ln|12000 - y| = k \cdot t + C_1$

$12000 - y = C_2 e^{-kt}$

$\underline{y = 12000 + C_3 e^{-kt}}$

$y(0) = 100 \Rightarrow \underline{\underline{y = 12000 - 11900 e^{-kt}}}$

c) $y(10) = 4000 \Rightarrow 4000 = 12000 - 11900 e^{-10k}$

$-10k = \ln\left(\frac{8000}{11900}\right)$

$\underline{\underline{K = 0{,}0397}}$

d) $6000 = 12000 - 11900 e^{-0{,}0397 \cdot t}$

$-0{,}0397t = \ln\left(\frac{6000}{11900}\right)$

$\underline{t = 17{,}24}$

Halvparten av innbyggerne er smittet etter $\underline{\underline{\text{ca. 17 uker}}}$

Oppgave 4

a) $P_1 = \frac{3 \cdot 1^2 - 1}{2} = \frac{2}{2} = 1$ Stemmer for $n=1$

Viser at formelen er riktig for $n+1$ dersom den er riktig for n:

$$P_{n+1} = P_n + 3n + 1 = \frac{3n^2 - n}{2} + 3n + 1$$

$$= \frac{3n^2 - n + 6n + 2}{2}$$

$$= \frac{3n^2 + 6n + 3 - (n+1)}{2}$$

$$= \frac{3(n^2 + 2n + 1) - (n+1)}{2}$$

$$= \frac{3(n+1)^2 - (n+1)}{2}$$

Dermed har jeg vist at formelen er rett for all $n \in \mathbb{N}$

Q.E.D.

b) Ser at $P_5 = T_5 + 2 \cdot T_4$ og mer generelt at $P_n = T_n + 2 \cdot T_{n-1}$

T_n er en aritmetisk rekke, altså;

$$T_n = \frac{n(a_1 + a_n)}{2} = \frac{n(1+n)}{2} \Rightarrow T_{n-1} = \frac{(n-1) \cdot n}{2}$$

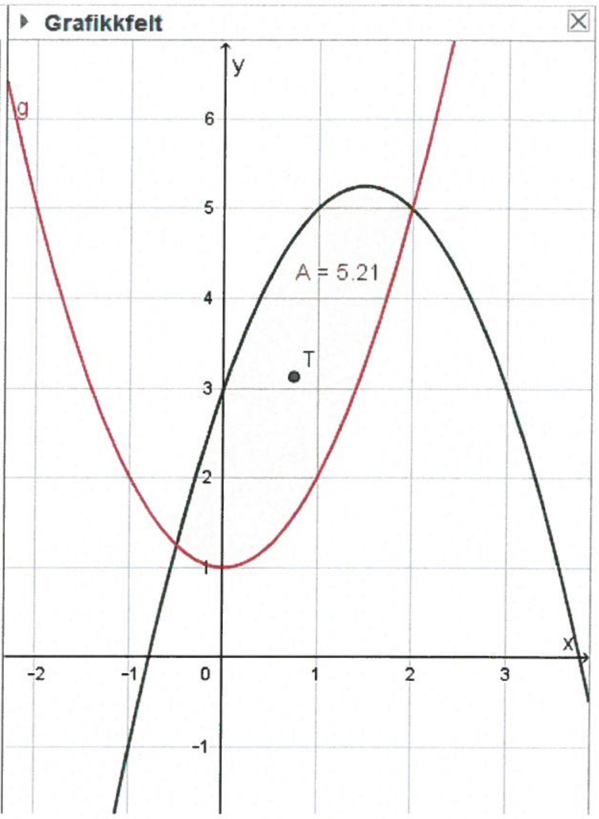
$$\Rightarrow P_n = \frac{n(1+n)}{2} + \frac{2 \cdot (n-1) \cdot n}{2} = \frac{n + n^2 + 2n^2 - 2n}{2} = \frac{3n^2 - n}{2}$$

VEDLEGG

Oppgave 1

CAS

1. Skjæring(f, g)
   → $\left\{(2,5), \left(-\frac{1}{2}, \frac{5}{4}\right)\right\}$
2. A:=IntegralMellom(f,g,(-1) / 2,2)
   → $A := \frac{125}{24}$
3. M:= Integral(x*(f-g), -1/2, 2)
   → $M := \frac{125}{32}$
4. N:=0.5*Integral(f^2-g^2, -1/2, 2)
   → $N := \frac{3125}{192}$
5. T := (M/A, N/A)
   → $T := \left(\frac{3}{4}, \frac{25}{8}\right)$

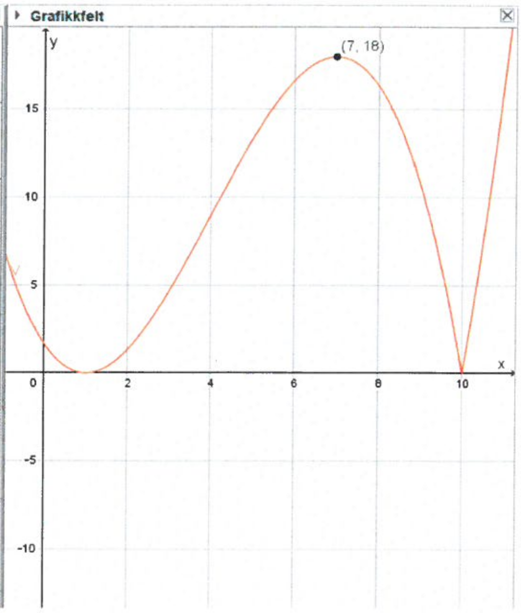


Se kommentarer i håndskreven besvarelse.

Oppgave 2

CAS

1. B(t) := (1, t+2, 3t)
   → $B(t) := (1, t+2, 3t)$
2. C(t) := (0, 4, t+1)
   → $C(t) := (0, 4, t+1)$
3. D(t) := (t-3, 8, 1)
   → $D(t) := (t-3, 8, 1)$
4. Areal(t) := 0.5*abs(Vektorprodukt(Vektor(A, B), Vektor(A, C)))
   → $\text{Areal}(t) := \frac{1}{2}\sqrt{t^4 - 18\,t^3 + 86\,t^2 - 34\,t + 21}$
5. Areal(2)
   → $\frac{13}{2}$
6. Areal = 6
   Løs: $\{t = -0.94, t = 1.8, t = 7.85, t = 9.29\}$
7. V(t) := (1/6)*abs(Skalarprodukt(Vektorprodukt(Vektor(A, B), Vektor(A, C)), Vektor(A, D)))
   → $V(t) := \frac{1}{6}\left|t^3 - 12\,t^2 + 21\,t - 10\right|$



Se kommentarer i håndskreven besvarelse.