Oppgave 1

a) $\frac{\sqrt{ab^4}\cdot\sqrt[3]{(ac)^6}}{\sqrt[4]{a^{10}}\cdot b^2} = \frac{(ab^4)^{\frac{1}{2}}\cdot((ac)^6)^{\frac{1}{3}}}{(a^{10})\cdot b^2} = \frac{a^{\frac{1}{2}}\cdot b^2\cdot(ac)^2}{a^{\frac{5}{2}}\cdot b^2} =$

$$a^{\frac{1}{2}+2-\frac{5}{2}} \cdot b^{2-2} \cdot c^2 = a^0 \cdot b^0 \cdot c^2 = \mathrm{c}^2$$

b) $2\ln(2a) - \ln(4a^2) = 2(ln2 \cdot lna) - (ln4 \cdot lna^2) =$

$2(ln2 \cdot lna) - (ln2^2 \cdot lna^2) = 2(ln2 \cdot lna) - 2(ln2 \cdot lna) = \underline{\underline{0}}$

c) $f(x) = 3x^4 + \cos(3x) - 5lnx$

$$f'(x) = 12x^3 + (-\sin(3x)) \cdot 3 - 5 \cdot \frac{1}{x}$$

$$= 12x^3 - 3\sin(3x) - \frac{5}{x}$$

d) $f(x) = e^{2x} lnx$

$$f'(x) = 2e^{2x} lnx + e^{2x} \cdot \frac{1}{x}$$

$$= e^{2x}\left(2lnx + \frac{1}{x}\right)$$

e) $f(x) = \frac{e^x}{x^2+3}$

$$f'(x) = \frac{e^x(x^2+3) - e^x 2x}{(x^2+3)^2}$$

$$= \frac{e^x(x^2 - 2x + 3)}{(x^2+3)^2}$$

f) $\int (x^4 + \sin(3x) - e^{2x})dx$

$$= \int x^4 dx + \int \sin(3x)\, dx - \int e^{2x} dx$$

$$= \frac{1}{5}x^5 + \left(-\frac{1}{3} \cdot \cos(3x)\right) - \frac{1}{2}e^{2x} + C$$

$$= \frac{1}{5}x^5 - \frac{1}{3}\cos(3x) - \frac{1}{2}e^{2x} + C$$

g) $\int x^6 e^{x^7} dx$

$$u = x^7$$

$$\frac{du}{dx} = 7x^6 \to \frac{du}{7x^6} = dx$$

$$\int x^6 \cdot e^u \cdot \frac{du}{7x^6}$$

$$\int \frac{e^u}{7} du = \frac{1}{7} e^u + C$$

$$= \frac{1}{7} e^{x^7} + C$$

Oppgave 2)

a) $\sqrt{2x+1} + 1 = x$

$$\sqrt{2x+1} = x - 1$$

$$Kvadrerer$$

$$\left(\sqrt{2x+1}\right)^2 = (x-1)^2$$

$$2x + 1 = x^2 - 2x + 1$$

$$x^2 - 4x = 0 \to x(x-4) = 0$$

$$gir\ x = 0\ eller\ x = 4$$

$$Setter\ inn\ og\ prøver$$

$$x = 0: \sqrt{2 \cdot 0 + 1} \neq 0\ eller$$

$$x = 4: \sqrt{2 \cdot 4 + 1} = 4 \to 3 + 1 = 4$$

$$gir\ løsning\ x = 4$$

b) $\sin(3x) = \frac{\sqrt{2}}{2} \qquad x \in [0, 2\pi >$

$$3x = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$3x = \frac{\pi}{4} + n \cdot 2\pi \; eller \; 3x = \frac{3}{4}\pi + n \cdot 2\pi$$

$$x = \frac{\pi}{4} + n \cdot 2\pi \mid \cdot \frac{1}{3} \; eller \; x = \frac{3}{4}\pi + n \cdot 2\pi \mid \cdot \frac{1}{3}$$

$$x = \frac{\pi}{12} + n \cdot \frac{2}{3}\pi \; eller \; x = \frac{\pi}{4} + n \cdot \frac{2}{3}\pi$$

$$L: \left\{\frac{\pi}{12}, \frac{\pi}{4}, \frac{3}{4}\pi, \frac{11}{12}\pi, \frac{17}{12}\pi, \frac{19}{12}\pi\right\}$$

c) $\frac{x+3}{x-2} \leq \frac{x+1}{x-3}$

$$x < 2, 3 < x \leq 7$$

Usikker

d) $5 + 25x + 125x^2 + 625x^3 + 3125x^4 + \cdots$

$$Rekka \; konvergerer \; når -1 < k < 1$$

$$\frac{25x}{5} = 5x = k$$

$$-1 < 5x < 1$$

$$-1 < 5x \; eller \; 5x < 1 \; for \; at \; rekken \; skal \; konvergere$$

$$-\frac{1}{5} < x \; eller \; x - \frac{1}{5} < x < \frac{1}{5}$$

$$S_n = \frac{1 - k^n}{1 - k} \cdot a_1 \rightarrow S_n = \frac{1 - 5x^n}{1 - 5x} \cdot 5$$

$$S_n = \frac{5 - 25x^n}{1 - 5x}$$

Usikker på svaret. Svaret gitt fra originale rekke og ikke konvergert rekke.

Oppgave 3)

f(x)=(x-1)e^x

a) $f(x) = 0 \rightarrow (x-1)e^x = 0$

$x - 1 = 0 \; eller \; e^x \neq 0$

$x = 1 \rightarrow NP: (1, 0)$

b) $f'(x) = e^x - e^x = e^x \cdot 1 + xe^x - e^x$

$= e^x(1 + x - 1) = e^x x$

$\rightarrow e^x x = 0 \rightarrow x = 0$

$fortegnsskjema \; gir \; bunnpunkt$

$BP: \big(0, f(0)\big) = (0, (-1 * e^{-1}) = (0, -1)$

c) $f'(x) = e^x x$

$f''(x) = e^x \cdot x + e^x = e^x(x+1)$

$f''(x) = 0 \rightarrow e^x(x+1) = 0$

$e^x \neq 0 \; eller \; x + 1 = 0 \rightarrow x = -1$

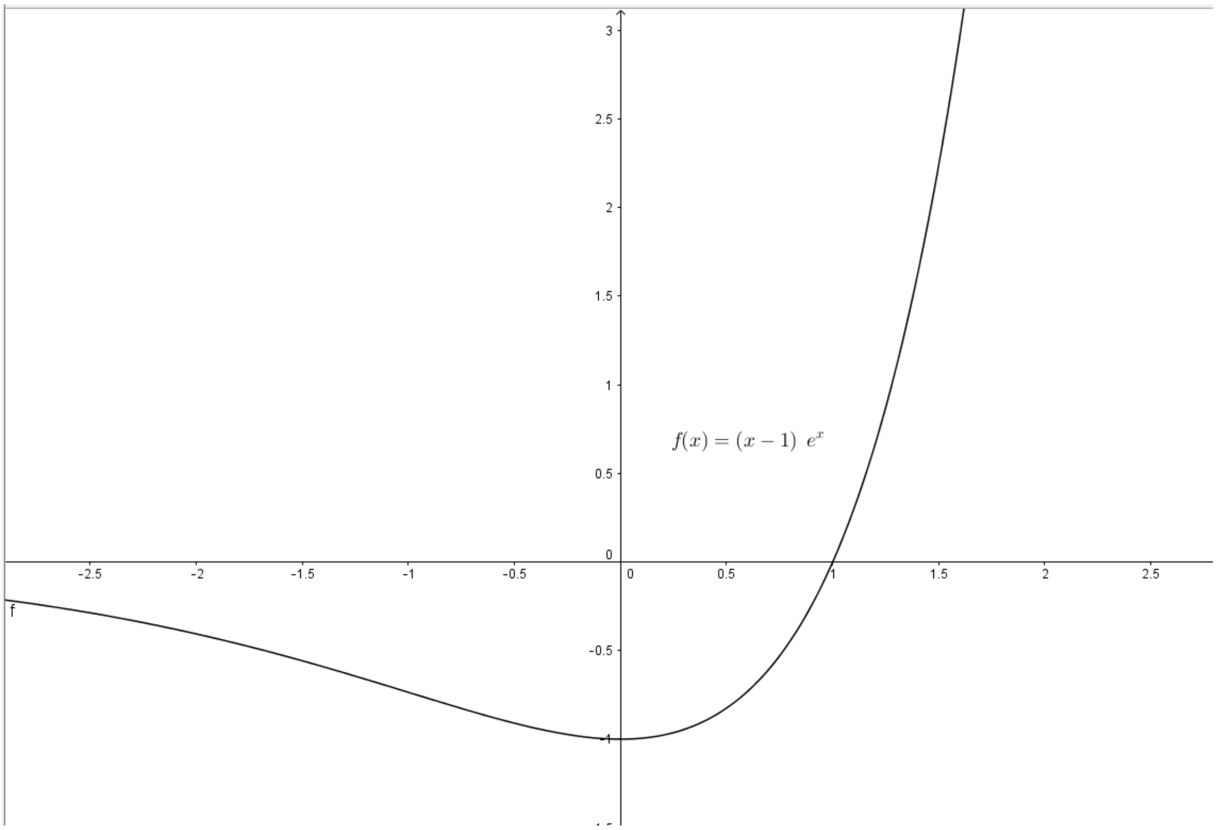
$VP: \big(-1, f(-1)\big) = (-1, -2e^{-1})$

d) $\big(1, f(1)\big) = (1{,}0) \rightarrow ettpunktsformelen$

$den \; deriverte \; til \; x = 1 \; gir \; stigningstallet \; a = e$

$y - 0 = e(x-1) \rightarrow y = ex - e$

e)



f) Areal: $\int_1^2 (x-1)e^x dx = 2{,}718$

Oppgave 4)

a) $\frac{15}{28} \approx 0{,}535 = 53{,}5\ \%\ for\ Gutt$

$$\frac{16}{28} = \frac{4}{7} \approx 0{,}517 = 57{,}1\ \%\ for\ Kaffe$$

b) $\frac{13}{28} * \frac{6}{13} = \frac{3}{14} \approx 0{,}214 = 21{,}4\ \%\ for\ J\ og\ K$

$$\frac{6}{13} \approx 0{,}462\ \%$$

Har egentlig ikke kontroll på sannsynlighet. Mulig det er feil..

Oppgave 5)

a) $\overrightarrow{AB} = [-2,-2,-2] \quad \overrightarrow{AC} = [-3,9,1]$

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = [-2,-2,-2] \cdot [-3,9,1] = (-2 \cdot -3, -2 \cdot 9, -2 \cdot 1)$$

$$= -14$$

$$|\overrightarrow{AB}| = \sqrt{(-2)^2 + (-2)^2 + (-2)^2} = 4$$

$$|\overrightarrow{AC}| = \sqrt{85}$$

$$< BAC = \cos^{-1}\left(\frac{-14}{\sqrt{6} * \sqrt{85}}\right) = 112{,}3°$$

b) $\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$$\frac{1}{2}\begin{vmatrix} -2 & -2 & -2 \\ -3 & 9 & 1 \end{vmatrix} = -2 - (-18), -(-2 - 6), -18 - 6$$

$$= \frac{1}{2}|[16, 8, -24]| = 4\sqrt{14} \approx 15$$

c) *Normalvektoren til planet gjennom A, B og C er representert ved*
$\overrightarrow{AB} \times \overrightarrow{AC} = [16, 8, -24]$ *som kan skrives som* $8[2, 1, -3]$
som gir normalvektoren $[2, 1, -3]$

d) $a(x - x_0) + b(y - y_0) + c(z - z_0) + d = 0$

Bruker punktet $B(0, -1, 0)$

$$2(x - 0) + 1(y + 1) - 3(z - 0) = 0$$

$$\alpha: 2x + y - 3z + 1 = 0$$

e) $l: \begin{cases} x = 1 + t \\ y = 3 - 8t \\ z = 1 + kt \end{cases}$

$$1 + t = 2 \bigwedge 3 - 8t = -5 \bigwedge 1 + kt = 0$$

$$t = 1 \;\rightarrow t = 1 \rightarrow k = -1$$

Oppgave 6)

a) $Skal\ lengden\ x\ bli\ et\ kvadrat\ må\ en\ side\ i\ kvadratet\ være\ \frac{x}{4}$

$$Areal\ av\ kvadrat: l \cdot b \rightarrow \frac{x}{4} \cdot \frac{x}{4} = \left(\frac{x}{4}\right)^2$$

$Omkretsenen\ til\ sirkelen\ vil\ være\ (1-x)$

$$1 - x = 2\pi r \rightarrow r = \frac{1-x}{2\pi}$$

$$Areal\ av\ sirkel = \ \pi r^2 \rightarrow \pi\left(\frac{1-x}{2\pi}\right)^2$$

$Da\ vil\ summen\ av\ de\ to\ arealene\ være$:

$$\left(\frac{x}{4}\right)^2 + \pi\left(\frac{1-x}{2\pi}\right)^2$$

b) $A(x) = \left(\frac{x}{2}\right)^2 + \pi\left(\frac{1-x}{2\pi}\right)^2 \quad x \in [0,1]$

$$A'(x) = 2\left(\frac{x}{4}\right) \cdot \frac{1}{4} + 2\pi\left(\frac{1-x}{2\pi}\right) \cdot \left(-\frac{1}{2\pi}\right)$$

$$= \frac{x}{8} - \left(\frac{1-x}{2\pi}\right)$$

$$A'(x) = 0 \rightarrow \left(\frac{\pi x - 4 + 4x}{8\pi}\right) = 0$$

$$\rightarrow \ \pi x - 4 + 4x = 0 \rightarrow \ \pi x + 4x = 4$$

$$x = \frac{4}{4+\pi} \rightarrow x = 0{,}56$$

$$A(0{,}56) = \left(\frac{0{,}56}{4}\right)^2 + \pi\left(\frac{1-0{,}56}{2\pi}\right)^2 = 0{,}035$$